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Probability Distribution in the Variability of the N_γ Factor of Shallow Foundations

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***Keywords*— Bearing Capacity, Bearing Capacity Factors, Friction Angle, Probability, Shallow Foundation.**

***Abstract*— Due to the existing variability in the calculation of the bearing capacity factors, specifically when obtaining the N_γ value of the bearing capacity, which is estimated by a large number of equations that have been obtained by approaches of limit equilibrium, method of characteristics, empirical, semi-empirical, among others; thus existing a great uncertainty, therefore, this article took a total of 55 equations that compute this value to carry out a probabilistic study where N_γ values were obtained for friction angles of 5º, 10º, 15º, 20º, 25º, 30º and 35º, being the variability obtained being directly proportional to friction angle. Also, it was found that the data have a good correlation with the Lognormal distribution, according to the Kolgomorov-Smirnov and Anderson-Darling tests, in this context, Lognormal cumulative probability distribution curves of the data under study were elaborated, thus allowing not to opt for a single or deterministic value but a probable or acceptable value according to the size of a project with a conservative range.*

I. INTRODUCTION

The soil plays an important role as a support base for any construction, the gravitational load transmissions of any structure go to this outcrop, where its load capacity indicates to the specialist a foundation geometry to choose from, and it is precisely that, in shallow foundations, bearing capacity generally governs the design process (Shahin & Cheung, 2011), representing a primary step for shallow foundation stability evaluations (Dewaikar et al., 2008).

For decades it has been known that bearing capacity has been studied and it has always been noted that it depends on certain factors; for example, without reaching various differentiated cases Terzaghi (1943) proposed an equation well known to date for the calculation of the bearing capacity considering the case of a strip footing, which is detailed below.

$$q_u = cN_c + qN_q + \frac{1}{2}\gamma B N_\gamma \qquad (1)$$

Where q_u is the ultimate bearing capacity of the soil, c is the cohesion, q is the overburden pressure, γ is the unit weight of soil, B is the width of footing, N_c, N_q and N_γ are the bearing capacity factors which depend on the friction angle ϕ of the soil.

From equation 1, is particularly see that the values of N_c and N_q were made known by Prandtl in 1921 and Reissner in 1924; however, there is a great dissertation on the value N_γ since this is theoretically less precise than the other two terms (Griffiths, 1982), so there is a wide controversy considered over the theoretical values of N_γ (Mahmood, 2018) thus leading to great variability in the objective of achieving the value of the bearing capacity by various methods (Padmini, 2018; Shill & Hoque, 2015; Sieffert & Bay-Gress, 2000), is so the bearing capacity must be better understood using new parametric and numerical analyzes (Sieffert & Bay-Gress, 2000), in this sense, today there are different investigations based on analytical model approaches, semi-empirical models, empirical models, finite difference models, upper limit and lower limit models, finite element models, etc. that try to determine this geotechnical parameter (Motra et al., 2016). As described, there are several proposed formulas, but no is totally accurate (Nguyen et al., 2016), generating as consequences discrepancies in the results (Ty et al., 2019).

The great geotechnical variability of the soil is well known, existing a certain proportion of uncertainty in the same parametric measurement, in this case, the N_γ value. Now, in this context, it is known that there are many forms of uncertainty as framed in JCGM (2008), where geotechnical uncertainty is recognized to be primarily epistemic, bayesian, and belief-based (Christian, 2004), of which this article will try to address the epistemic in a probabilistic way related to 55 formulas taken from Motra et al. (2016) that obtain the N_γ value, thus trying to improve the knowledge of this parameter, generating a reduction in epistemic uncertainties and therefore the total variability of the estimated soil design properties are reduced, as mentioned in Cao et al. (2017).

Characterization of geological uncertainty remains challenging (Juang et al., 2019), and it is particularly knowing that the model to choose is still the main source of uncertainty (Motra et al., 2016), due to geotechnical properties vary spatially (Han et al., 2020; Juang et al., 2019; Popescu et al., 2005). However it is known that probabilistic methods provide a powerful tool for dealing with uncertainty in engineering projects (Christian, 2004), it is thus that what this article raises is to deal in a probabilistic way the value N_γ of bearing capacity under different friction angles ϕ, considering the results through a Cumulative Distribution Function (CDF), in this context, it will be allowed not only to opt for a single deterministic value but by a range of probabilistic values.

II. METHODS

2.1. Model for prediction bearing capacity factors.

Although equation 1 is used in practice to calculate the bearing capacity of shallow foundations; however, to obtain the value N_γ, influencing the bearing capacity there are a large number of methods. Table 1 shows 55 models to compute the N_γ value, which have been taken from the compilation of Motra et al. (2016).

Table 1: Models for prediction N_γ value adapted from Motra et al. (2016)

Nº	Author	Method	Model
1	Terzaghi (1943)	Limit equilibrium	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+3.0\right]\tan(1.34\phi)$
2	Taylor (1948)	Limit equilibrium	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$
3	Caquot and Kérisel (1953)	Method of characteristics	$N_\gamma = \left[1.413\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.794\right]\tan(1.27\phi)$
4	Biarez et al. (1961)	Equilibrium limit	$N_\gamma = 1.8\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan\phi$
5	Feda (1961)	Empirical	$N_\gamma = 0.01\exp\left(\frac{\phi}{4}\right)$ (for $\phi < 35^\circ$, ϕ in degree)
6	Meyerhof (1963)	Semi-empirical based on limit equilibrium	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.4\phi)$
7	Hu (1964)	Fitted model, equilibrium limit	$N_\gamma = \left[1.901\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+0.27\right]\tan(1.285\phi)$
8	Krizek (1965)	Empirical	$N_\gamma = \frac{6\phi}{40-\phi}$(for $\phi < 35^\circ$, ϕ in degree)

Table 1: Models for prediction N_γ value adapted from Motra et al. (2016) *(continued)*

Nº	Author	Method	Model
9	Booker (1969)	Method of characteristics	$N_\gamma = 0.1045\exp(9.6\phi)$
10	Hansen and Christensen (1969)	Fitted model, Method of characteristics	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.33\phi)$
11	Muhs and Weiss (1969)	Semi-empirical model	$N_\gamma = 2.0\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan\phi$
12	Abdul-Baki and Beik (1970)	Fitted model, limit equilibrium	$N_\gamma = \left[1.752\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+0.186\right]\tan(1.32\phi)$
13	Brinch-Hansen (1970)	Semi-empirical	$N_\gamma = 1.5\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan\phi$
14	Davis and Booker (1971)	Fitted model, equilibrium limit	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+2.33\right]\tan(1.316\phi)$
15	Chummar (1972)	Fitted model, semi-empirical	$N_\gamma = \left[7.12\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+65.5\right]\tan(0.27\phi)$
16	Vesic (1973)	Method of characteristics	$N_\gamma = 2.0\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right]\tan\phi$
17	Chen (1975a)	Upper bound limit analysis	$N_\gamma = 2.0\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right]\tan\phi\tan\left(\frac{\pi}{4}+\frac{\phi}{5}\right)$
18	Chen (1975b)	Fitted model, upper bound limit analysis	$N_\gamma = \left[1.45\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+0.754\right]\tan(1.41\phi)$
19	Salenon et al. (1976)	Fitted model, limit equilibrium	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.405\phi)$
20	Craig and Pariti (1978)	Fitted model, limit equilibrium	$N_\gamma = \left[2.22\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+0.222\right]\tan\phi$
21	Spangler and Handy (1982)	Approximation from Terzaghi's mechanism	$N_\gamma = 1.1\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.3\phi)$
22	Ingra and Baecher (1983)	Statistical analysis	$N_\gamma = \exp(0.173\phi-1.646)$ (ϕ in degree)
23	Simone and Restaino (1984)	Method of characteristics	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.341\phi)$
24	Hettler and Gudehus (1988)	Empirical	$N_\gamma = \exp\left[5.7(\tan\phi)^{1.15}\right]-1.0$
25	Saran and Agarwal (1991)	Limit equilibrium	$N_\gamma = \exp\left(\frac{0.757}{\ln\phi}+15.286\phi-3.452\right)$
26	Bolton and Lau (1993a)	Method of characteristics	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.5\phi)$

Table 1: Models for prediction N_γ *value adapted from* Motra et al. (2016) *(continued)*

Nº	Author	Method	Model
27	Bolton and Lau (1993b)	Fitted model, method of characteristics	$N_\gamma = \left[1.274\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+3.736\right]\tan(1.367\phi)$
28	Kumbhojkar (1993)	Fitted model, numerical solution	$N_\gamma = \left[1.2\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.324\right]\tan(1.417\phi)$
29	Zadroga (1994)	Empirical model	$N_\gamma = 0.657\exp(0.141\phi)$ (ϕ in degree)
30	Manoharan and Dasgupta (1995)	Fitted model finite element	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+3.464\right]\tan(1.279\phi)$
31	Bowles (1996)	Fitted model from K_{py} values	$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{py}}{\cos\phi}-1.0\right),\ K_{py} = \exp\left(1.708+3.287\phi-\frac{0.34}{\ln\phi}\right)$
32	Frydman and Burd (1997)	Fitted model, finite difference analysis	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right]\tan(1.4\phi)$
33	Michalowski (1997)	Upper bound limit analysis	$N_\gamma = \exp(0.66+5.11\tan\phi)\tan\phi$
34	Paolucci and Pecker (1997)	Fitted model, upper bound limit analysis	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right]\tan(1.71\phi)$
35	Danish Standards Association (1998)	Empirical fitting	$N_\gamma = \frac{1}{4}\left\{\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\cos\phi\right\}^{1.5}$
36	Soubra (1999)	Fitted model, upper bound analysis	$N_\gamma = \left[1.374\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-0.162\right]\tan(1.343\phi)$
37	Coduto (2001)	Approximation from Terzaghi's model	$N_\gamma = \frac{\left[1.374\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right]\tan\phi}{1.0+0.4\sin(4.0\phi)}$
38	Poulos et al. (2001)	Solution based on Davis and Booker (1971)	$N_\gamma = 0.1054\exp(9.6\phi)$
39	Ueno et al. (2001)	Fitted model, method of characteristics	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.436\phi)$
40	Wang et al. (2001a)	Fitted model one, upper bound limit analysis	$N_\gamma = 1.2\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+4.6\right]\tan(1.436\phi)$
41	Wang et al. (2001b)	Fitted model two, upper bound limit analysis	$N_\gamma = \left[1.234\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+4.151\right]\tan(1.394\phi)$
42	Zhu et al. (2001a)	Case 1, limit equilibrium	$N_\gamma = \left[2.0\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right](\tan\phi)^{1.35}$
43	Zhu et al. (2001b)	Case 2, limit equilibrium	$N_\gamma = \left[2.0\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right]\tan(1.07\phi)$

Table 1: Models for prediction N_γ adapted from Motra et al. (2016) *(continued)*

Nº	Author	Method	Model
44	Dewaikar and Mohapatra (2003)	Fitted model, limit equilibrium based on Terzaghi's model	$N_\gamma = \left[1.626\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+2.019\right]\tan(1.373\phi)$
45	Kumar (2003a)	Fitted model, method of characteristics	$N_\gamma = \left[0.96\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+0.508\right]\tan(1.352\phi)$
46	Kumar (2003b)	Fitted model, upper bound analysis	$N_\gamma = \left[1.379\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-0.461\right]\tan(1.337\phi)$
47	Hjiaj et al. (2005)	Lower and upper bound analysis	$N_\gamma = \exp\left[\frac{\pi}{6}(1+3\pi\tan\phi)\right](\tan\phi)^{\frac{2\pi}{5}}$
48	Martin (2005)	Fitted model, method of characteristics	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.338\phi)$
49	Smith (2005)	Method of characteristics	$N_\gamma = 1.75\left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp\left[(0.75\pi+\phi)\tan\phi\right]-1.0\right]\tan\phi$
50	Kumar and Kouzer (2007)	Lower and upper bound limit analysis	$N_\gamma = \left[1.012\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-0.226\right]\tan(1.426\phi)$
51	Lyamin et al. (2007)	Lower and upper bound	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-0.6\right]\tan(1.33\phi)$
52	Kumar and Khatri (2008)	Fitted model, lower bound finite element linear programming	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.26\phi)$
53	Salgado (2008)	Approximation model from N_γ values of Martin (2005)	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)-1.0\right]\tan(1.32\phi)$
54	Yang and Yang (2008)	Fitted model, upper bound limit analysis	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right]\tan(1.396\phi)$
55	Jahanandish et al. (2010)	Fitted model, zero extension lines method	$N_\gamma = \left[\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\exp(\pi\tan\phi)+1.0\right]\tan(1.5\phi)$

As can be seen from the previous table, there is a great variety of models to obtain the N_γ value. This is due to the methodology used by the authors, the considerations and assumptions made such as the one usually used in which it is considered that the soil is a rigid material and that it generally fails due to shear (Meyerhof, 1951), and also due to the premise that soils may be homogeneous in terms of composition, they may not be homogeneous in terms of mechanical behavior (Uzielli et al., 2007).

2.2. Variability of prediction models.

It is known that geotechnics is not an exact science (Lacasse & Nadim, 1998) and that the specific geotechnical parameters remain an open question in its analysis (Cao et al., 2016), this is clearly reflected in the large number of formulas presented in Table 1, that is why these formulas have been grouped for friction angles of 5º, 10º, 15º, 20º, 25º, 30º y 35º; the value of up to 35º has been chosen because some formulas establish an approach of ϕ <35º e.g. Krizek (1965). The result of the variability of the N_γ values can be seen in the following figures.

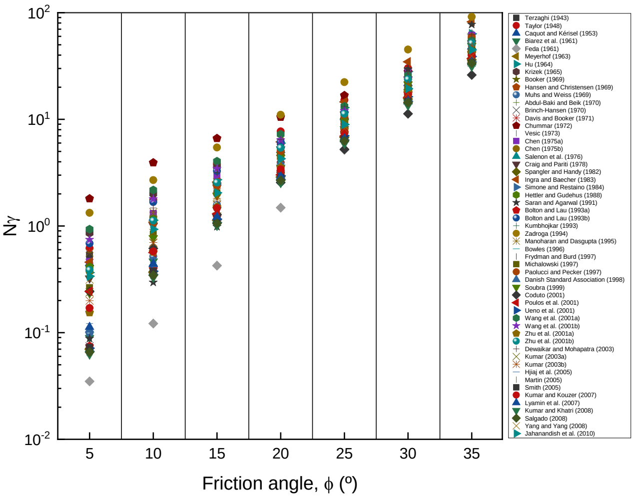

Fig. 1: Availability of the N_γ values for each author

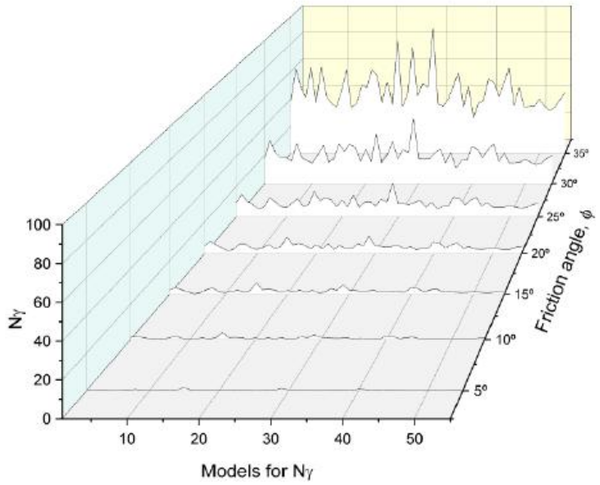

Fig. 2: Variability of the N_γ values for the 55 models in the study

From Fig. 2 shown, it is easy to check what is established in Padmini (2018) where the bearing capacity factor N_γ varies sharply with friction angle, especially when it is on the rise, thus Table 2 denotes both the maximum and minimum N_γ values for the studied database, where there is clearly a differentiation of values of up to 65.33 (specifically in ϕ =35º). Also, it is shown in Fig. 3 that the average values and the average values $\pm 1\sigma$ (standard deviation) for different friction angles, where most of the estimated models (ordered according to Table 1) are within this range, reflecting the existence of similarity of values between model and model, but when wanting to consider a unique estimated value of N_γ, this becomes somewhat complicated. Although different countries already have predefined different methods used to estimate the N_γ factor as shown in Sieffert & Bay-Gress (2000), it may be that these values used in a deterministic way do not exactly define or characterize the soil profile for different geotechnical zones, as it is widely known that the variability of soil properties encountered in any project is related to the particular site and specific regional geology (Baecher & Christian, 2003), so it is convenient to see this problem through a statistical approach. Therefore, if we expand the methodology in a method of probabilities based on uncertainties, we will have the advantage of providing more complete and realistic information regarding the level of safety of design (Uzielli et al., 2007), finding not a true value, but an acceptable or conforming value, to the reality of the safety of any project.

Table 2: Maximum and minimum values of N_γ

ϕ (º)	**Maximum and minimum values of N_γ**			
	Maximum	**Author**	**Minimum**	**Author**
5	1.81	Chummar (1972)	0.03	Feda (1961)
10	3.92	Chummar (1972)	0.12	Feda (1961)
15	6.62	Chummar (1972)	0.43	Feda (1961)
20	11.02	Zadrogra (1994)	1.48	Feda (1961)
25	22.31	Zadrogra (1994)	5.18	Feda (1961)
30	45.15	Zadrogra (1994)	11.27	Coduto (2001)
35	91.37	Zadrogra (1994)	26.04	Coduto (2001)

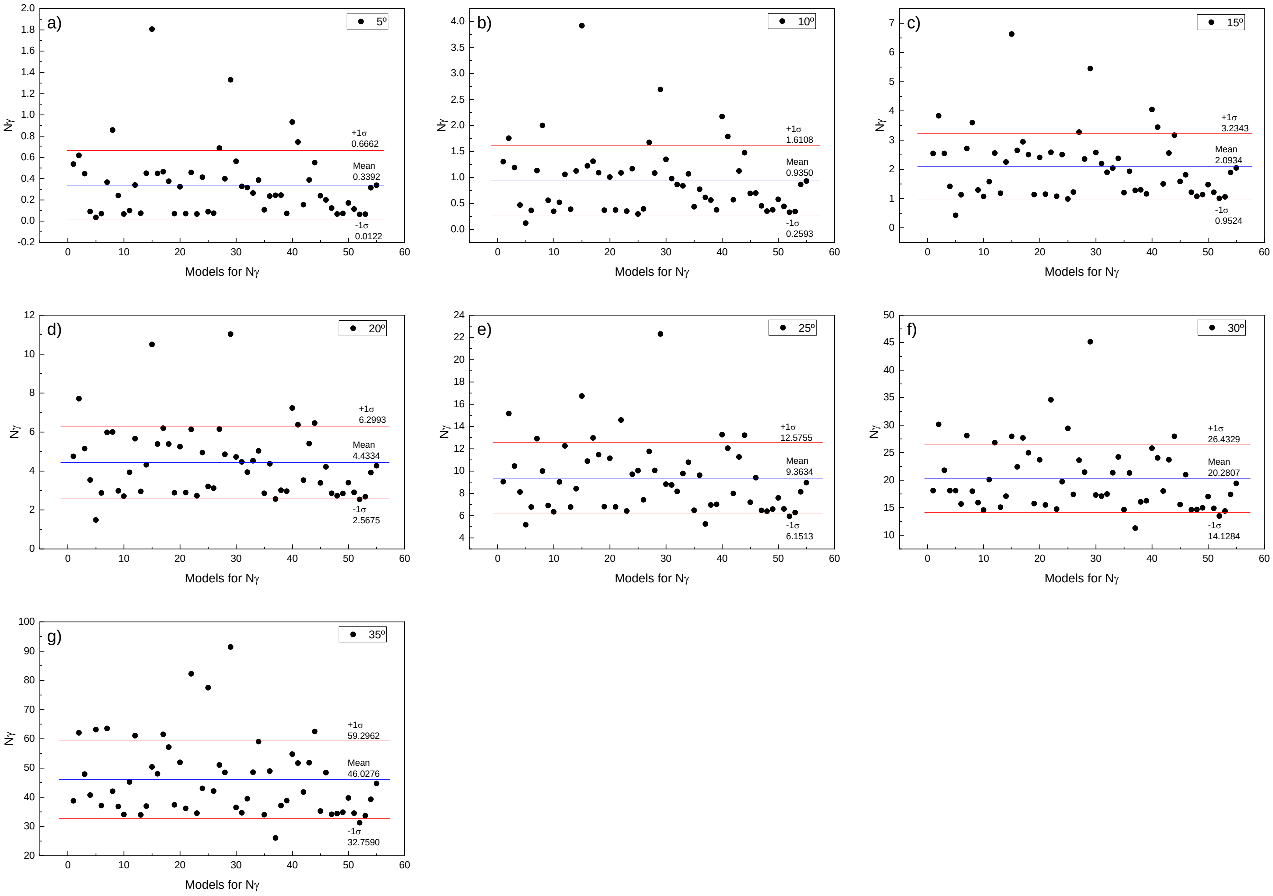

Fig. 3: Range of values $\pm 1\sigma$ for friction angles: a) 5º, b) 10º, c) 15º, d) 20º, e) 25º, f) 30º and g) 35º

2.3. Probabilistic approach.

The use of probability models and statistical methods to analyze data has become common practice in virtually all scientific disciplines (Devore, 2008), many studies that are based on probabilistic terms about geotechnics and soils have been developed in recent years (Al-Bittar & Soubra, 2013; Chen et al., 2012; Griffiths et al., 2002; Jung et al., 2008; Shahin & Cheung, 2011). If we see, for example, in a general approach, for engineering, the most common continuous probability distribution functions are the Exponential, Gamma, Beta, Uniform, Weibull, Rayleigh, Normal and Lognormal. On the other hand, in a slightly more particular approach, in geotechnics, the distributions used in the literature to model soil properties are the Lognormal, Gamma and Beta (Chen et al., 2012; Popescu et al., 2005). In this context, some examples are cited such as Vessia et al. (2009), who assumed in his study a Lognormal distribution, and Han et al. (2020) used the Beta, Gamma and Lognormal distributions.

2.4. Probabilistic function.

The probability function used in this article is the Lognormal. This distribution showed a good correlation with the estimated N_γ values for the seven friction angles under study, which could be verified using both the Kolgomorov-Smirnov test (Massey, 1951) and Anderson-Darling test (Rahman et al., 2006) to ensure the goodness of fit of the distribution. The results are shown in Table 3.

The Probability Density Function (PDF) of the Lognormal distribution is given as:

$$f(x;\mu,\sigma)=\frac{1}{\sqrt{2\pi}\sigma_{\ln}x}\exp\left[-\frac{1}{2}\left(\frac{\ln(x)-\mu_{\ln}}{\sigma_{\ln}}\right)^2\right] \qquad (2)$$

Also, the probability value associated with the Cumulative Distribution Function (CDF) is as follows:

$$P\left[X\le x\right]=\Phi\left(\frac{\ln(x)-\mu_{\ln}}{\sigma_{\ln}}\right) \qquad (3)$$

Where $\mu_{\ln}$ and $\sigma_{\ln}$ are the mean and standard deviation of $\ln(x)$ respectively.

Equation 2 was used to match the probability distribution presented by the N_γ value at different friction angles. It was found that the variability of the data has a good correlation with the Lognormal distribution as shown in Fig. 4, where the data of the N_γ values for the different friction angles was plotted on the X-axis and the value on the Y-axis Lognormal probability.

Table 3: Tests for the estimation of N_γ probability Lognormal distribution

ϕ (º)	Critical significance level	Testing				Decision
		Kolmogorov-Smirnov test		Anderson-Darling test		
		Critical value	Statistics	Critical value	Statistics	
5	0.05	0.17981	0.12815	2.5018	1.24130	The hypothesis should not be rejected
10	0.05	0.17981	0.09717	2.5018	0.72867	The hypothesis should not be rejected
15	0.05	0.17981	0.12073	2.5018	0.89589	The hypothesis should not be rejected
20	0.05	0.17981	0.11905	2.5018	0.80133	The hypothesis should not be rejected
25	0.05	0.17981	0.11198	2.5018	0.57770	The hypothesis should not be rejected
30	0.05	0.17981	0.15610	2.5018	0.93521	The hypothesis should not be rejected
35	0.05	0.17981	0.11978	2.5018	1.06750	The hypothesis should not be rejected

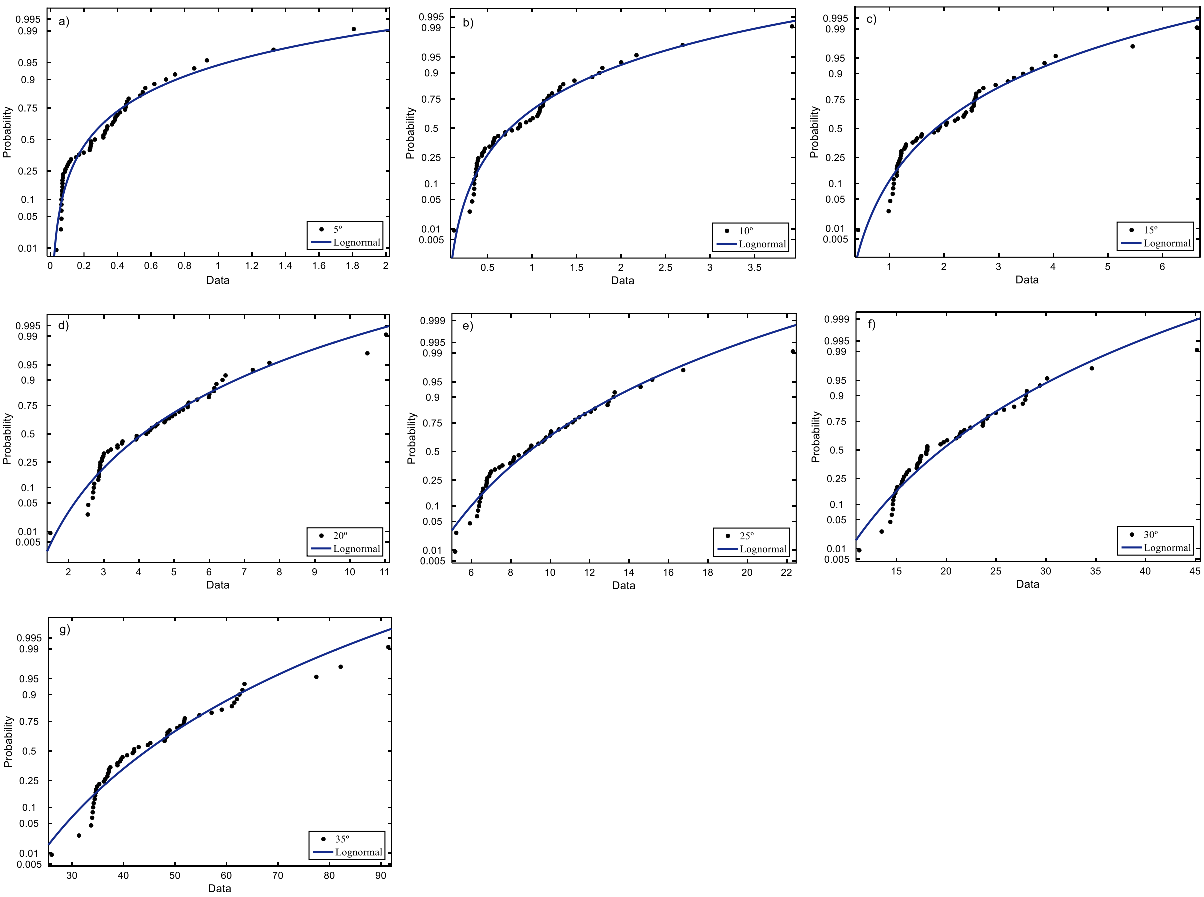

Fig. 4: Lognormal probability distribution curves for N_γ values for friction angles: a) 5º, b) 10º, c) 15º, d) 20º, e) 25º, f) 30º and g) 35º

III. RESULTS

Based on the good correlation that exists between the Lognormal probability distribution, the data under study shown in the previous section and using equation 3 for the CDF, seven probability distribution curves were obtained for the friction angles 5º, 10º, 15º, 20º 25º, 30º y 35º, which are shown in Fig. 5.

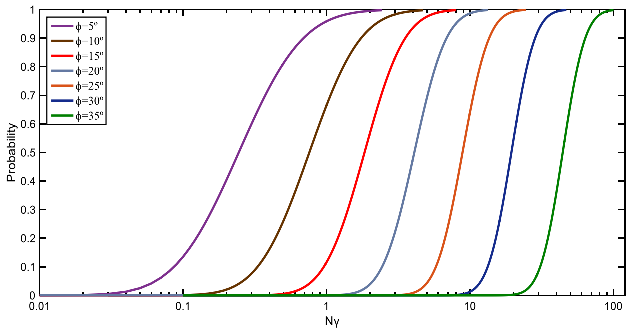

Fig. 5: Lognormal CDF of N_γ values for different friction angles

IV. DISCUSSION

The curves shown in Fig.5 are commonly used to express probabilities of damage under a certain level of demand (Ahmed & Soubra, 2012; Popescu et al., 2005) but they are also used to express probability levels of different parameters as in Vessia et al. (2009) who used it in the dataset of bearing capacity of strip foundation or Chen et al. (2012) who used it in undrained shear strength. In this context, these curves allow access not only to select a single value, that is, to a characteristic value of the parameter N_γ; instead, select a value based on probabilities, under certain reliability that the user considers relevant, thus allowing to reduce the uncertainty when choosing which appropriate equation best estimates this parameter.

V. CONCLUSION

In this article, the probability distribution of 55 approaches by various authors was investigated to obtain the N_γ value of the bearing capacity under the friction angles: 5º, 10º, 15º, 20º, 25º, 30º y 35º. The results are shown as follows.

There is great variability in obtaining the N_γ values through the 55 equations proposed by various authors, finding that this variation is proportional to the friction angle; that is, the greater the friction angle, the greater the variability of the estimated value of N_γ and vice versa.

Also, the N_γ values calculated for the friction angles mentioned above follow a suitable Lognormal probability distribution.

Finally, faced with the problems encountered in engineering, specifically concerning soils, it is very common to use a deterministic practical approach; however, in parameters with great dissertation such as the N_γ value, key in the bearing capacity, it is not advisable to choose a unique value knowing that there are currently different methodologies to calculate, in that sense, by incorporating previously verified CDF, in obtaining this parameter, it was found that the results are shown using a probabilistic approach reduce bias and uncertainty to characterize the value of bearing capacity, generating in decision making opt not only for a deterministic value based on a single formula but to choose one or more probable or appropriate values to the conservative quality required in a project.

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